SIMILARITY SOLUTIONS FOR THE POWER LAW FLUIDS FLOW BEHIND A FLAT PLATE

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ABSTRACT. In this note the power law fluids flow in the wake behind a flat plate, placed in the direction of a uniform stream, has been discussed for similar solutions. Two group of transformations have been used and it is found that for similarity purposes the velocity distributions in the wake at $y = 0$ are of the forms (i) $w(x, 0) = C_1 x^{\pm\frac{1}{2}}$, (*ii*) $w(x, 0) = C_2\, e^{[(n+1)/(n-1)]qx}$, where C_1 and C_2 are certain constants in terms of the boundary conditions, n is flollow behaviour index, $w(x, y) = U - u(x, y)$, where u is the velocity along the plate and U Is the free stream velocit.y

The boundary layer equations for two dimensional flow along the plate may be taken as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial}{\partial y}\left(\left| \frac{\partial u}{\partial y} \right|^{-1} \frac{\partial u}{\partial y} \right) \qquad \ldots \quad (1.1)$$

where n is the flow behaviour index for the power law ffiuids flow, ν the coefficient of kinematic viscosity, u and v are the velocity components along and perpendicular to the direction of the flow. The flat plate is considered to lie along the x-axis and the flow is considered along the x-axis with $u = U$ outside the boundary layer. The velocity distribution may be calculated far down stream from the plate where the velocity difference (1)

$$w(x, y) = U - u(x, y) \qquad \ldots \quad (1.2)$$

is small so that the boundary layer equation may be further simplified in order to get some asymptotic law for the wake.

Substituting (1.2) into (1.1) and neglecting the quadratic terms in w, we have the equation of boundary layer flow

$$u\,\frac{\partial w}{\partial x} = (-1)^{n-1}\,\nu\,\frac{\partial}{\partial y}\left(\left| \frac{\partial w}{\partial y} \right|^{-1} \frac{\partial w}{\partial y} \right) \qquad \ldots \quad (1.3)$$

with boundary conditions

$$y = 0 : \frac{\partial w}{\partial y} = 0; \; y = \infty : w = 0(u = U) \qquad \ldots \quad (1.4)$$

The first boundary condition implies that at $y = 0$, w = constant.

LINEAR GROUP TRANSFORMATION

Recently the author (Lal, 1967) has used group theoretical methods for similarity purposes. These are extended in this note for similar solutions. We put into (1.3)

$$x = A^{\alpha_1} \bar{x}, \; y = A^{\alpha_2} \bar{y}, \; w = A^{\alpha_3} \bar{w} \qquad \ldots \quad (2.1)$$

where α_1, α_2, α_3 and A are certain constants. Thus for invariance of the transformations, we have

$$\alpha_3 - \alpha_1 = n(\alpha_3 - \alpha_2) - \alpha_2 \qquad \ldots \quad (2.2)$$

or,

$$b(n+1) - m(n-1) = 1 \qquad \ldots \quad (2.3)$$

or,

$$b = \frac{1+(n-1)m}{n+1} \qquad \ldots \quad (2.4)$$

where

$$b = \frac{\alpha_2}{\alpha_1}, \quad m = \frac{\alpha_3}{\alpha_1} \qquad \ldots \quad (2.5)$$

Using above values in (2.1), we have

$$\left.\begin{aligned} \frac{y}{x^b} &= \frac{\bar{y}}{\bar{x}^b} \\ \frac{w}{x^m} &= \frac{\bar{w}}{\bar{x}^m} \end{aligned}\right] \qquad \ldots \quad (2.6)$$

Therefore the absolute invariants are

$$\eta = \frac{y}{x^{[1+m(n-1)]/(n+1)}} \qquad \ldots \quad (2.7)$$

$$g(\eta) = \frac{w}{x^m} \qquad \ldots \quad (2.8)$$

where m is an arbitrary constant. Substituting (2.7) and (2.8) into (1.3), we have

$$\frac{(-1)^{n-1}\nu}{U}\frac{d}{d\eta}\left(\left|\frac{dg}{d\eta}\right|^{n-1}\frac{dg}{d\eta}\right)+\frac{1+m(n-1)}{n+1}\eta\frac{dg}{d\eta}-mg=0 \qquad \text{--- (2.9)}$$

Thus the similar solution exists, if

$$w=C_1x^m \qquad \text{... (2.10)}$$

where C_1 is an arbitrary constant.

The boundary conditions are reduced to

$$\left.\begin{aligned}\eta=0:\ g&=C_1\\ \eta=\infty:\ g&=0\end{aligned}\right] \qquad \text{... (2.11)}$$

However if we introduce

$$\left.\begin{aligned}\eta&=\frac{y}{x^b}\sqrt{\frac{U}{\nu}}\,e^{-\frac{\pi}{\;}(n-1)},\\ w&=Ux^{-\frac{1}{2}}g(\eta),\end{aligned}\right] \qquad \text{... (2.12)}$$

into (1.3), we have in place of (2.9) that

$$\frac{d}{d\eta}\left(\left|\frac{dg}{d\eta}\right|^{n-1}\frac{dg}{d\eta}\right)+\frac{1+m(\eta-1)}{n+1}\eta\frac{dg}{d\eta}+mg=0 \qquad \text{... (2.13)}$$

which for the Newtonian fluid flow ($n=1$) is reduced to

$$\frac{d^2g}{d\eta^2}+\frac{n}{2}\frac{dg}{d\eta}+\frac{g}{2}=0 \qquad \text{... (2.14)}$$

whose asymptotic solution is known (Pai, 1956) as

$$g=e^{-\eta^2/4} \qquad \text{... (2.15)}$$

SPIRAL GROUP TRANSFORMATION

We put into (1.3)

$$x=\bar{x}+\beta_1b,\ y=e^{\beta_2b}\bar{y},\ w=e^{\beta_3b}\bar{w} \qquad \text{... (3.1)}$$

where β_1, β_2, β_3 and b are certain constants.

Thus for invariance of the transformations

$$\beta_3 = n\beta_3 - (n+1)\beta_2 \qquad \dots \quad (3.2)$$

For $n = 1, \beta_2 = 0$.

The absolute invariants for this group of transformations are obtained from,

$$\frac{w}{e^{px}} = \frac{\bar{w}}{e^{p\bar{x}}}, \quad p = \beta_3/\beta_1 \qquad \dots \quad (3.3)$$

Thus

$$w = G(y)e^{px} \qquad \dots \quad (3.4)$$

and from (3.4) and (1.3), we have

$$\frac{d^2G}{dy^2} - pUG = 0 \qquad \dots \quad (3.5)$$

whose solution is well known and discussed for large and small value of p as well for the oscillatory motion by putting $p = i\omega$, where ω is the frequency of fluctuations.

For $n \neq 1$, we have

$$\beta_3(n-1) = (n+1)\beta_2 \qquad \dots \quad (3.6)$$

Thus

$$\frac{\beta_3}{\beta_1} = \frac{n+1}{n-1}\frac{\beta_2}{\beta_1} = \frac{n+1}{n-1}q \qquad \dots \quad (3.7)$$

and we have

$$\frac{w}{e[(n+1/(n-1)]} = \frac{\bar{w}}{e[(n+1(/(n-1)]q\bar{x}} \qquad \dots \quad (38)$$

$$\frac{y}{e^{qx}} = \frac{\bar{y}}{e^{q\bar{x}}} \qquad \dots \quad (3.9)$$

Thus, we take for the absolute invariance,

$$\zeta = \frac{y}{e^{qx}}$$

$$H(\zeta) = \frac{w}{e[(n+1)/(n-1)]q\bar{x}} \qquad \dots \quad (3.10)$$

From (3.10 and (1.3), we have

$$\frac{(-1)^{n-1}\nu}{U}\frac{d}{d\zeta}\left(\left|\frac{dH}{d\zeta}\right|^{n-1}\frac{dH}{d\zeta}\right)+\zeta q\frac{dH}{d\zeta}-\frac{(n+1)}{(n-1)}qH=0 \quad \ldots \quad (3.11$$

Thus the conclusion is that for similar solutions, we have

$$w(x, 0) = C_2\, e^{[(n+1)/(n-1)]qx} \quad \ldots \quad (3.12)$$

where $C_2 = H(0)$.

REFERENCES

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